Dynamic Meinongian logic for descriptions

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Abstract

The aim of this article is to present a formal language and a semantics for it which reflect two ideas expressed but not formally implemented in Graham Priest's *Towards Non-Being*. One is that a description can be evaluated in different ways depending on the speaker's intention, so that one and the same statement can be true on one evaluation and be false on the other. The other idea is that two different kinds of linguistic activities are involved in fiction, as well as in mathematics: characterising an object and inferring about the object thereby characterised. The language defined here is designed so as to conform to the ideas as naturally as possible. The resulting language and its semantics is Meinongian in that it assign every term a denotation, but also Russellian in that it treat existing objects as specially privileged.

1 Introduction

There are two radically opposing kinds of approaches to determining the meanings of those sentences which contain descriptions (*description sentences* hereafter): Meinongian and Russellian. In a Meinongian approach, a description is treated as a term which refers to an objects, just as a name does. However, since some descriptions, say "the present King of France," seem not to refer to any existent object, it should be possible for descriptions (or terms in general) to refer to non-existent objects. We will call such objects "non-beings." Non-beings are just another kind of genuine objects, capable of having a variety of properties though they lack the property of existence. On the other hand, in a

Russellian approach, descriptions are not integral grammatical units in themselves and have no referents at all. Therefore we need not worry about whom (what) "the present King of France" refers to. Thus, these two approaches stand in a sharp contrast as for fundamental semantic principles.[1]

For Russell, all genuine individual objects are existent. For Meinong, the range of genuine objects includes abstract entities like numbers, fictional objects like Sharlock Holmes, or even logically impossible objects like a circle that is not round, in addition to concrete existent objects.

An obvious advantage of Meinongianians is that they can save the intuition that a sentence like "Sharlock Holmes is a detective" is true. This is due to the "Characterisation Principle (Postulate)" (CP), which Meinongians will in general embrace. Meinongians think that fictional objects, non-existent objects can be identified by *characterisation.* For example, Sharlock Holmes is identified by all the properties that Doyle attributed to Holmes in his novels. Such descriptions as "the present King of France" attributes the property of being the present King of France to the object it refers to. So descriptions are also characterisation. The Characterisation Principle states that

> (CP) Non-existent objects have those properties which they are characterised as having, and those which follows from them.

In virtue of this, we can attribute various properties to non-beings.

However, CP has serious difficulties as well. For example, consider the referent of the description "the circle that is not round." It follows from CP that the object is round and not round. Since Russell pointed out this difficulty, several attempts have been made to resolve it. Among them, Terence Parsons [7] has been perhaps most prominent. His idea was to discriminate those descriptions which refer from those which do not. For this purpose, he proposed several criteria for a description to refer. However such criteria cannot help being arbitrary and highly artificial.

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[1] Yet another view concerning description sentences is Strawson's, which treat any sentence containing an empty description as being neither true or false (Strawson [13]). Though seemingly quite opposing, Strawson's and Russell's views are similar in that they treat all the sentences containing empty descriptions as not false, no matter what the sentences say. So we can apply our method to both of the theories. See §3.

In *Towards Non-Being* (TNB, hereafter), Priest builds a semantics based on Meinongian theory of objects, which he calls *noneism* following Richard Routley (Routley [9]), and thereby resolves the difficulties of CP, as well as some puzzles in traditional epistemic logics. His approach does not only resolve puzzles, but also is a natural analysis of our practice of speaking about fictional objects or mathematical objects. Moreover the semantics has a merit of being able to treat every intentional verbs in a single apparatus, instead of treating exclusively "know" or "believe." On the other hand, TNB's approach does not seem to deal well with typically Russellian cases like "The present King of France is bald." Yet, Priest suggests informally how to deal with those cases. In short the idea is to shift the methods of evaluation of description sentences according to the speaker's intention. It is the aim of this paper to formally implement this idea. At the same time, we also take into consideration another suggestion by Prist that two different kinds of activities are involved in fiction, as well as mathematics: characterising an object and discussing the object characterised.

2 TNB's approach to description sentences

In this section, we will see how descriptions are dealt with in TNB, how their meanings are determined, and compare its merits and demerits with those of the Russelian approach.

2.1 The world semantics

First we will review the semantic framework of TNB. [2] The semantics is a variation of possible world semantics with a constant domain, where there are impossible and open worlds in addition to usual possible worlds. This is hence called *world semantics*. Impossible worlds are those worlds where some of logical truth might not hold. Open worlds are those worlds which might not be closed under logical consequences. Namely, an open world w are such that there might be propositions $A_1, A_2, \ldots, A_n, B$ such that B is a logical consequence of $A_1, A_2, \ldots, A_n$, $A_1, A_2, \ldots, A_n$ are true in w, but B is not. The actual world @

[2] The following is a fairly simplified version of the semantics of TNB, omitting some aspects irrelevant to the current issue.

is among the possible ones. We will denote possible, impossible and open worlds by $\mathcal{P}$, $\mathcal{I}$, and $\mathcal{O}$ respectively. The worlds in $\mathcal{P} \cup \mathcal{I}$ are called closed worlds, written $\mathcal{C}$. $\mathcal{C} \cup \mathcal{O}$ is written $\mathcal{W}$. Given an assignment s of an object to each of the variables, we write $w \Vdash_s A$ to express that A is true at w under the assignment s.[3]

The domain of the objects is common to all the worlds, denoted by D. D contains both existent and non-existent objects in a world. There is no essential distinction between these objects. "Exists" is no more or no less an ordinary predicate than predicates like "is red" or " is a human" are. Existent (non-existent) objects may differ from world to world. Holmes does not exist in our actual world but he does in some other worlds. This is why we need not variable domains.

One purpose of TNB is to provide a semantics for verbs expressing propositional attitudes like "believes that A." This kind of verbs are called *intentional operators*. Let a be a term, Ψ an intentional operator and A a formula. Then the formula $a\Psi A$ stands for the sentence "a Ψs that A." In the following, we need not consider the subject a, and we will omit it and write "ΨA" and so on. In TNB, intentional operators are treated as a kind of necessity modal operators. That is, relative to each intentional operator Ψ, there is an accessibility relation R_Ψ such that

$$w \Vdash_s \Psi A \iff \text{for all } w' \in \mathcal{W} \text{ if } wR_\Psi w' \text{ then } w' \Vdash_s A.$$

The worlds accessible from w by R_Ψ are the ones where every proposition toward which the subject has the attitude Ψ in w is true. For example, if Ψ stands for believing, the worlds accessible from w by R_Ψ are the worlds where whatever the subject believes in w is true.

This is standard in traditionla epistemic logic.[4] What is unique in TNB is the introduction of impossible and closed worlds so as to block some of problematic inferences such as *Logical Omniscience* etc.[5] But we are not concerned with them here.

[3] s is needed because A may contain free variables. When it is the case, its truth can not be decided unless we have assigned an object to each variable as its denotation.

[4] For standard semantics in epistemic logic, see, for example, Hintikka [3] or Meyer and van der Hoek [6].

[5] A similar approach can be found in Hintikka [4].

2.2 The Meinongian account of descriptions

What is most characteristic in the Meinongian account of descriptions is that, unlike the Russelian account, it regards a description as a well-formed term never failing to refer to an object in the domain. It is natural to ask the following questions:

1. What do seemingly empty descriptions such as "the gold mountain" refer to?
2. How is the Characterisation Principle satisfied?
3. Is it possible to avoid the paradox of CP?

Let's see the answers to these questions in TNB.

As for the question 1. A description describes a certain condition and thereby picks up an object which satisfies the condition. If it is intended that only one object satisfies the condition, the description is called *definite* and otherwise *indefinite*. In a formal language, the conditions is expressed in the form of logical formula. The object denoted by a description is that which satisfies the formula. Therefore, a description can be constructed by designating a formula and a variable to be satisfied. Following TNB notation, we will write ιxA (or εxA) for the definite (or indefinite) description determined by a formula A and a variable x. If we are indifferent to the definite/indefinite distinction, we will write ξxA instead. We will call A in ξxA the *characterisation* or *characterising property* of the description.

TNB assigns an object to every description. The assigned object is called the denotation of the description. If the characterisation is satisfiable in the actual world, then it has as its denotation an object that satisfies the characterisation in the actual world. Therefore, each description ξxA has a selection function $\phi_{\xi xA} : \mathcal{P}(D) \to D$. Let $S = \{d \in D : @ \Vdash_{s(x\mapsto d)} A\}$, where for any objects $d_1, d_2, \ldots, d_n \in D$ and any variables$x_1, x_2, \ldots, x_n$, $s(x_1 \mapsto d_1, \ldots, x_n \mapsto d_n)$ is the assignment defined by

$$s(x_1 \mapsto d_1, \ldots, x_n \mapsto d_n)(v) = \begin{cases} d_i & \text{if } v = x_i \ (1 \le i \le n) \\ s(v) & \text{otherwise} \end{cases} ,$$

where $x_i \neq x_j$ for every $0 \le i, j \le n$ such that $i \neq j$. In other words, S is the set of objects that satisfy the characterisation in the actual world. When S is not empty, it is postulated that $\phi_{\xi x A}(S) \in S$. In this paper, we will call such descritpions *realistic*. When S is empty, $\phi_{\xi x A}(S)$ is not restricted, and can be any term in D. we will call such descritpion *idealistic*.

A realistic description denotes an object that satisfies the characterisation in the actual world, and the truth of a sentence containing the description generally depends on actual properties of the denoted object. For example, the sentence "The present president of France is bald" is about the present president of France in the actual world and may be true or false depending on the actual fact about the object. On the other hand, although a idealistic description does denotes a certain object, it does not satisfy the characterisation in the actual world. It is the objects which the speaker indends to satisfy the characterisation.

It is natural to question what kind of objects idealistic objects like the present King of France are. Of course they are not physical objects. Nor are they mental, for if so, it wiould be entirely impossible for two persons to talk about one and the same non-being. But we can talk about the same Sharlock Holmes or Zeus with others.[6]

As for the question 2. In relation to descriptions, the Characterisation Principle may be interpreted as stating that for any variable x and any formula A

$$\models A[x := \xi x A],$$

where $A[x := \xi x A]$ stands for the result of substituting $\xi x A$ for free occurrences of x in A. Of course this is not always the case. For example, let A be the predicate "x is a gold mountain." Then CP requires that something be a gold mountain in this actual world.[7] But there is nothing that is actually a gold mountain.

Priest regards this as an inappropriate way to interpret CP concerning descriptions. To say something about a characterised object is related to an intentional propositional attitude of representing a certain situation. When someone says "The gold mountain is a gold mountain" one is not stating it as a fact about this world, but a fact about the situation (or the possible world) that one

[6]See TNB chapter 5 for the identity of non-beings.

[7]The logical consequence relation $\models$ is defined as the truth preservation in the actual world for every interpretation. Therefore $\models A \iff$ for any variable assignment s, $@ \Vdash_s A$ holds in every model.

is representing using the description "the gold mountain." If we use Φ for the intentional operator "represents," CP actually requires that for any x and A

$$\models \Phi A[x := \xi x A].^{8}$$

Interpreted this way, the constraint causes no trouble in the semantics of TNB for this says only that $A[x := \xi x A]$ is true in every world where everything that the speaker represents is true.

This account applies to other kinds of characterisation than descriptions. For example, Sharlock Holmes has as its characterisation the whole body of the declarative sentences in the Holmes series written by Doyle. By CP, Holmes satisfies the characterisation (being detective, living in the Baker Street etc.). This is, however, not the case in the actual world but in the world where everything Doyle represented in the Holmes series holds true.

Thus TNB makes CP completely general without any exception, by relativising it to the worlds connected to the actual one by the intentional operator "represents."

As for the question 3. If CP holds true without exceptions, it appears that the paradoxes mentioned above are inevitable. It indeed is, in a sense. Take the description "a circle that is not a circle" as an example. Let Px be a predicate "x is a circle." Then "a circle that is not a circle" is written $\varepsilon x(Px \wedge \neg Px)$. Let τ be the description. By CP $\models \Phi(Px \wedge \neg Px)[x := \tau]$, namely $\models \Phi(P\tau \wedge \neg P\tau)$. This means that in any interpretation, $@ \Vdash \Phi(P\tau \wedge \neg P\tau)$ where $@$ is the actual world and Φ is the intentional operator for "represents." Then for any world $w \in \mathcal{W}$, if $@R_{\Phi}w$ then $w \Vdash P\tau \wedge \neg P\tau$. Therefore, both $P\tau$ and $\neg P\tau$ hold true in any world that is Φ-accessible from the actual world $@$. But it only means that such worlds are impossible worlds (therefore not the actuall world in particular). It does not matter if any contradiction arises in an impossible world.

2.3 Types of descriptions

However, this analysis does not seem to apply to every description sentence. Consider, for example, the sentence "The planet which is nearer to the sun

[8] Recall we decided to forget about who has that propositional attitude.

	Speaker's intention	Referent
(1)	To represent the reality	Existent
(2)	To represent the reality	Non-existent
(3)	To represent a fictional situation	Existent
(4)	To represent a fictional situation	Non-existent

Table 1: Four types of descriptions

than Mercury causes its perihelion shift" uttered by an astronomer. Call the sentence S. According to the above approach, S is interpreted as speaking about the worlds represented by the astronomer in which there is a planet nearer to the sun than Mercury. If the worlds are represented as obeying ordinary laws of physics, the planet will have an effect on the orbit of Mercury in any of those worlds. So S will be true in such worlds. Is this, however, correct analysis of the utterance? The astronomer would not have represented the planet as something existing in his imagination but as something existing in this actual world. So the sentence should not interpreted as stating something about the worlds in which his representation is realized, but stating something about this actual world. In this case, the description sentence should not be relativised to the represented (non-actual) worlds.

Another kind of cases to which the above approach does not apply are the ones where the speaker uses description intending to represent a fictinal object but there exists an object satisfying the characterisation. For example, suppose that someone uses the descriptions "the present emperor of Japan" not knowing that there is one in Japan. According to TNB, "the present emperor of Japan" has as its referent the actual present emperor of Japan. On the other hand, the object intended by the description has properties such as being imagined by the speaker, which the actual emperor does not have. This seems problematic.

When we use descriptions, there seem to be two kinds of cases: on the one hand, the speaker intends to represent some fictional situation, while, on the other hand, the speaker intends to refer to some actual object. In addition, descriptions may or may not have their referents in the actual world. Then we can classify the cases where a description is used as shown in the table 1.

	Russellian	Meinongian
(1)	✓	✓
(2)	✓	×
(3)	×	×
(4)	×	✓

Table 2: Russell versus Meinong

Russellian approaches intend to deal well with the cases (1) and (2), and not with (3) and (4). On the other hand, Meinongian approaches are supposed to deal with the cases (1) and (4), and not with (2) and (3), as shown in the table 2.

Priest does not give a detailed account for the case (2), but says that it is possible that we use a description which no existent object fits, intending to apply it to the actual world. He suggested that we have different kinds of representing. Namely, representing something as fictional, and representing something as real. In the latter case, the associated accessibility relation will be reflexive, i.e., relate the the actual world to itself. Then $@ \Vdash \Phi A$ implies $@ \Vdash A$. Therefore, $@ \nVdash A$ implies $@ \nVdash \Phi A$.

As for the case (3), Priest, metioning the possibility that a certain existent object may satify by chance characterising conditions of a fictional character, and hence the writer of the fiction may be talking about the object contrary to his intention, says as follows:

> But when I tell a work of fiction, I deliberately intend to exclude this possibility. Thus, we should perhaps rule out this world as a candidate for satisfying it. It therefore makes sense to suppose that the appropriate intentional state involved in representing things in this case, Φ', is different from that, Φ, in which I intend the stroy to be veridical. We may take $a\Phi' A$ to be something like: 'a represents A as holding non-actually (in the matter at hand)'. (Priest [8] p. 124)

In contrast to the case (2), this time the associated accessibility relation should be modified so that it will not relate the actual world to itself. Moreover, the referent of the description should be different from the usual one. This is because the imagined object may have properties which any actual object does not, say being imagined by the author at such and such a moment and so on.

What this means is that (syntactically) the same description can refer to different objects depending on the speaker's intention or the situation in which it is used. This is intuitively obvious. If someone is talking about *Dr. Strangelove* and says "the president of the United States," then he refers to the faint-hearted man with eye glasses in the film whose part Peter Sellers played, and not to Barack Obama.

In TNB, different ways in which the speaker represents the object may be reflected by different models. Here we will provide an alternative approach to address the problem of the speaker's intention.

3 Dynamic Meinongian logic

In TNB, Priest makes another important point, though he does not formally implement it. He thinks that our activities concerning fiction and mathematics are very much alike, and that for both of them, there are two distinct kinds of activities: 'specifying a characterization' on the one hand, and 'figuring out what follows from it' on the other (Priest [8] p. 148). Yet when thinking about fiction, one naturally thinks of the former, while when thinking about mathematics, one thinks of the latter. And this explains why fiction and mathematics are recognized as totally different kinds of activities.

The former activity, i.e., the activity of specifying a characterisation, is not explicitly dealt with in Priest [8] except for the characterisation by descriptions. Here we try to define the notion of characterisation in general, in the way that it will cover descriptions as well. Then we make up a semantics for characterisation which reflects the intuition (pointed out in the previous section) that the way of evaluating a description (or a name of a characterised object) can differ according to the speaker's intention.

In the following, $\mathbf{V}$ stands for the set of variables, $\mathbf{C}$ for that of constants, $\mathbf{S}$ for that of formulas in the first-order language. In addition, we write $\mathbf{S}^*$ for the set of all the finite sequence on $\mathbf{S}$. For simplicity, we consider a language without function symbols. For $A \in \mathbf{S}$, $FV(A)$ is the set of free variables occuring in A. We fix a interpretation $\mathcal{I}$ for this language.

We extend the language with following expressions.

Definition 3.1 (Characterising declaration; discourse)**.** For any formula A and variable x, $\langle A, x\rangle$ is called a *characterising declaration.* Beside usual well-formed formulas, CA is also a wwf if C is a characterising declaration and A is a wwf. Let $C_1, C_2, \ldots, C_n$ be characterising declarations and A be a formula. Then we call $C_1C_2\ldots C_nA$ a *discourse.*

A characterising decration is not evaluated on its own. It functions by affecting the evaluation of the formula that comes after it in a discourse. In this sense it looks like a propositional operator such as $\Box$.

We shall give a semantics for this language as follows. We denote by D the domain of the objects.

Definition 3.2 (Characterising context)**.** A function from $\mathbf{V}$ to $\mathbf{S}^*$ is called a *characterising context.* Given a characterising context $\chi : \mathbf{V} \to \mathbf{S}^*$, a variable x and a sequence of formulas $\mathbf{s}$, $\chi_{(x\mapsto\mathbf{s})}$ stands for the characterising context defined by

$$\chi_{(x\mapsto\mathbf{s})(y)} = \begin{cases} \mathbf{s} & \text{if } x = y \\ \chi(y) & \text{otherwise} \end{cases}$$

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Definition 3.3 (denotations of variables)**.** For any variable x, $\phi_x : P(D) \to D \cup \{\perp\}$ is a function such that $\phi_x(M) \in M$ if M is not empty, and $\phi_x(\emptyset) = \perp$, where $\perp \notin D$.

For any formula A and assignment $s : \mathbf{V} \to D$, define $\sigma^A_{s,\mathcal{I}} : \mathbf{V} \to \mathcal{P}(D)$ by

$$\sigma^A_{s,\mathcal{I}}(x) = \{d \in D : \mathcal{I} \models_{s(x\mapsto d)} A\} \quad (x \in \mathbf{V}).$$

Furthermore, for a given characterising context χ, define $\sigma^\chi_{s,\mathcal{I}} : \mathbf{V} \to \mathcal{P}(D)$ by

$$\sigma^\chi_{s,\mathcal{I}}(x) = \bigcap_{A\in\chi(x)} \sigma^A_s(x) \quad (x \in \mathbf{V}).$$

Then a denoting function $[\![\cdot]\!]^\chi_{s,\mathcal{I}} : \mathbf{V} \to D \cup \{\perp\}$ is defined by

$$[\![x]\!]^\chi_{s,\mathcal{I}} = \phi_x(\sigma^\chi_s(x)) \quad (x \in \mathbf{V}).$$

We call $[\![x]\!]^{\chi}_{s,\mathcal{I}}$ the *denotation* of x (relative to the interpretation $\mathcal{I}$, the assignment s and the characterising context χ). The subscript $\mathcal{I}$ may be omitted if no confusion arises.

We extend the evaluation of formulas so as to deal with the cases in which some of the variables have as their value $\bot \notin D$. Let s be a function from $\mathbf{V}$ to $D \cup \{\bot\}$. For any atomic formula $Px_1 \ldots x_n$, if $s(x_i) = \bot$ for some $1 \leq i \leq n$, then $\mathcal{I}_s(Px_1 \ldots x_n) = \bot$. Otherwise, $\mathcal{I}_s(Px_1 \ldots x_n)$ is either true or false as usual. For a complex formula A, if $\mathcal{I}_s(B) = \bot$ for a subformula B of A, then $\mathcal{I}_s(A) = \bot$. Otherwise $\mathcal{I}_s(A)$ is true or false as usual. We can see $\mathcal{I}_s$ as a partial function from $\mathbf{S}$ to $\{true, false\}$, as opposed to a total function that $\mathcal{I}_s$ usually is.

Definition 3.4 (local logics)**.** Any subset $\vdash$ of $\mathbf{S}^* \times \mathbf{S}^*$ is called a *local logic* on $\mathbf{S}$.

A local logic is a logic in some particular piece of fiction or a theory. It may seem natural to impose some constraints on local logics, such as reflexivity or monotonicity. You can do so and in many cases you should. However, we can think of a story where monotonicity, say, does not hold. Therefore we leave local logics without any constraint.

Definition 3.5 (evaluation, Meinongian and Russellian)**.** Given a local logic $\vdash$ and an assignment $s : \mathbf{V} \to D$ of variables. Let $\mathbf{x} \subseteq_{\text{fin}} \mathbf{V}$. Then an *evaluation* $\Vdash^s_{\mathbf{x}}$ is defined for any characterising context χ and formula A by

$$\chi \Vdash^s_{\mathbf{x}} A \iff \begin{cases} \bigcup_{x \in \mathbf{x}} \chi(x) \vdash A & \text{if } \mathbf{x} \neq \varnothing \\ \mathcal{I} \models_{[\![\cdot]\!]^{\chi}_{s}} A & \text{otherwise.} \end{cases}$$

An evaluation is called *Meinongian* if $\mathbf{x} \neq \varnothing$, and *Russellian* otherwise.

Intuitively, what $\chi \Vdash^s_{\mathbf{x}} A$ states is that, when we regard the set of variables $\mathbf{x}$ as names for Meinongian objects and/or "objectives," we can infer A from their characterising properties according to the local logic $\vdash$. If $\mathbf{x}$ is empty, it says that the denotations of the free variables in A satisfy A under the interpretation $\mathcal{I}$. Thus, in the latter case, the variables are dealt with like Russelian descriptions. When any of the variables has $\bot$ as its denotation, A is false, which conforms to the Russellian treatment of description of which the denotation does not exist.

Note that we can treat (definite) descriptions and fictional proper names in the same manner. They are both variables associated with a particular set of formulas. Therefore "Holmes" and "Pegasus" are not constants. We reserve constatns as names for existent objects. Therefore, we cannot characterise the things that exist.

We have not yet given an account on how characterisation will be carried out. An act of characterisation will be carried out by using a characterising declaration. A characterising declaration $\langle A, x \rangle$ has the effect of altering a given characterising context χ. To be more specific, $\langle A, x \rangle$ will add A to the set of characterising properties of x. Formally, this is defined as follows.

Definition 3.6 (dynamic evaluation)**.** Consider characterising declarations $C_i = \langle A_i, x_i \rangle$ $(1 \leq i \leq n)$ and a characterising context χ. For a variable assignment s and a finite set of variables $\mathbf{x}$, the *dynamic evaluation* $\chi \Vdash^{*s}_{\mathbf{x}} C_1 C_2 \ldots C_n A$ *of* $C_1 C_2 \ldots C_n A$ is defined as follows:

(1) if $n = 0$, $\chi \Vdash^{*s}_{\mathbf{x}} A \iff \chi \Vdash^{s}_{\mathbf{x}} A$, and

(2) if $n \geq 1$, $\chi \Vdash^{*s}_{\mathbf{x}} C_1 C_2 \ldots C_n A \iff \chi' \Vdash^{*s(x_1 \mapsto [\![x_1]\!]^{\chi'}_s)}_{\mathbf{x}} C_2 \ldots C_n A$, where $\chi' = \chi_{(x_1 \mapsto \langle A_1, \chi(x_1) \rangle)}$.

As mentioned above, Priest distinguishes two kinds of linguistic activities concerning fiction and mathematics, namely, the activity of designating the characterisation and that of explicating what follows from the characterisation. Each C_i in the discourse $C_1 C_2 \ldots C_n A$ corresponds to the act of designating the characterisation, while A to the conclusion from the characterisation. As to fiction, the former corresponds to the creation of a work of fiction, while the latter to the discussion of the fiction (particularly, argumentation about what is entailed from it). As to mathematics, the former corresponds to postulation of axioms, while the latter to statement of theorems that follow from the axioms.

This approach can be applied not only to fiction and mathematics, but also to cases where one uses a description intending to represent the reality. For example, some 19th century astronomers thought that there was a planet nearer to the sun than Mercury, and it causes its perihelion shift. They often called this imagined planet "Vulcan." They did not make up the story as a piece of fiction. They intended the name to refer to some existing object. Therefore the statement "Vulcan causes Mercury's perihelion shift" is false. This intuition is reflected in our system as follows:

Let $C_1 = \langle x \text{ is a planet}, x \rangle$,

$C_2 = \langle x \text{ is nearer to the sun than Mercury}, x \rangle$,

$A = x$ causes Mercury's perihelion shift, and

$\chi(z) = \varnothing$, for any variable z.

Then the Russellian evaluation $\chi \Vdash^s_\varnothing C_1C_2A$ is false, while the Meinongian evaluation $\chi \Vdash^s_x C_1C_2A$ is true under a proper local logic $\vdash$.

Another kind of cases to consider are the ones in which the speaker intends to represent a fictional situation but the characterisation happens to hold true of some existent object. Then the object will have unintended properties, and the semantics of TNB allows us to infer such properties about that object. On the other hand, in our system you can signify your intention by indicating variables that are meant to refer to Meinongian objects. Then you cannot infer anything other than those that can be deduced from the characterising properties. This situation might seem strange, but it is in fact common when we turn our eye to the practice of mathematics, as we will see in §3.1

3.1 Consideration

Two opposing intuitions concerning description sentences led to two quite different semantic accounts of them, namely, Meinongian and Russellian. The difference lies in whether we regard a description as refering to something (not necessarily existent) or being merely quantificational and not referential.[9] Our everyday discourse involves instances which lend support to either of the two intuitions, and so, over a hundred years, researchers have tended to appeal to some of them and expressed preference for one theory over the other. However, neither of them alone seems to be able to deal with all the instances where

[9] In this sense, Strawson's view is similar to the Meinongian view. Meanwhile, he thought that the cases where the descriptions fail to refer are anomalies, and the sentences containing such descriptions cannot be true. In this respect, Strawson is more similar to Russell, who thought that in such cases the sentences are always false, than to Meinong, who thought that such sentences can be either true or false.

descriptions are used. If so, all we can say is that there are some cases where Russell scores over Meinong, and there are other cases where Meinong beats Russell, as shown in the table 2. Therefore, here we did not try to judge which theory is better and instead build a system which accomodate these two opposite way to evaluate description sentences.

One merit of our system is that it reflects two distinct kind of linguisitic activities involved in fiction as well as mathematics. Namely characterising an object (writing a work of fiction or postulating axioms), and drawing conclusions about from the characterisation (making a guess about fictional situations, or deducing theorems from a set of axioms). In a discourse $C_1C_2 \ldots C_nA$, $C_1C_2 \ldots C_n$ corresponds to the former, and A to the latter.

This analysis will be of some interest for the philosophers of mathematics. We observed in the previous section that we may use the Meinongian evaluation for descriptions for which there are existent objects that satisfy their characterising properties. In such a case, we refrain from ascribing any property that is not deduced from the characterising properties. This is what happens when a mathematician make a new axiomatic system abstracted from some existent theories. For example, when Dedekind came up with the theory of lattice, he has in mind a lattice of submodules of a module over a ring. Such a lattice has more properties than are deduced from the lattice axioms.[10] Such generalization is important because it allows us to convey any result obtained from the axioms to wider range of mathematical structures such as Boolean algebras and so on.[11]

Our system has another merit of not allowing arbitrary conclusions to be derived from axioms which no mathematical structrures can satisfy. For even for such axiomatic systems, we have a Meinongian object which satisfy the axioms, and so we do not have to be worried about vacuity, which has concerned many mathematical structuralists.[12]

[10]Among others, it has the property of being *modular*.

[11]For the history of lattice theory, see Schlimm [10].

[12]Cf. Hellman [2].

4 Conclusion

In this article, we presented a formal system which reflects the two intuition expressed in Priest [8]: One is that a description can be evaluated in different ways depending on the speaker's intention. The other idea is that two different kinds of linguistic activities are involved in fiction and in mathematics.

In our everyday discourse we can find instances of description sentences which favour or challenge either of the two method of evaluation: Meinongian and Russellian. We saw that in the resulting system, we can carry out both Russelian and Meinongian evaluations of description sentence, so that we can make up for the defects of these methods. Furthermore, our system is provide a plausible model for mathematical activities.

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